Sample Midterm Questions

Shading and Interpolation

(Q1) Suppose you are given a triangle in 2D with vertices at positions (0, 0), (1, 0) and (0, 1). In the following, assume the positions of these vertices are expressed with homogeneous column vectors as $a = [0, 0, 1]^T$, $b = [1, 0, 1]^T$, and $c = [0, 1, 1]^T$, using an orthonormal basis in which the first unit-length basis vector points to the right and the second points up.

1. Give a transformation matrix that translates this triangle by 5 units to the left.
2. Give a transformation matrix that rotates this triangle counterclockwise by 90 degrees.
3. Give a transformation matrix that translates this triangle by 5 units to the left and also rotates by 90 degrees counter-clockwise about its own origin (that is, about point a in its original coordinate system).
4. Suppose under some affine transformation T the triangle is mapped to $a' = [4, 2, 1]$, $b' = [2, 4, 1]$ and c' = [4, 4, 1]. Find the matrix that represents this transformation.

(Q2) If a line is being drawn with RGB = (180, 200, 10) at end point A and RGB = (90, 260, 40) at end point B, what will be the RGB value of a point C that is between A and B and is twice as far from A as B? Assume that the line is being drawn under parallel projection and the colors are computed using linear interpolation.

Shading and Lighting

(Q3) It has been suggested that the ambient term in shading model can be replaced by a light source at the eye point. That is, instead of just adding in ambient illumination, a new light source at the eye is added with no attenuation, and is treated just like the other light sources in the scene. How is this approach similar to using ambient illumination? How is it different? Consider both raytracing and the polygon rendering engines common in graphics hardware.

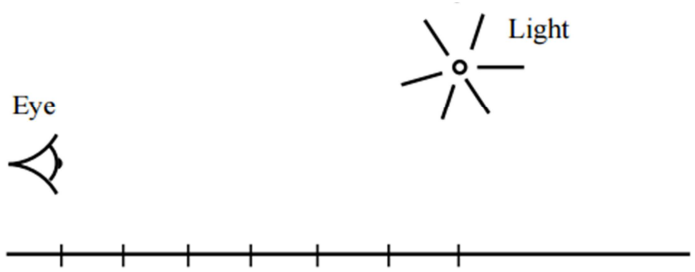
(Q4) In our illumination model, there were three terms: an ambient term, a diffuse term, and a specular term.

What effect does the ambient term model?

What assumption(s) do we make for the diffuse term?

In the specular term, there is a factor of $(r.v)^{\alpha}$. What effect does varying the α power have?

(Q5) In the 2D figure below, we have the eye located on the left, a plane (line) along the bottom, and a point light source towards the right; the light is twice as far from the plane/line as the eye is from the plane/line. The marked and labeled locations on the plane/line indicate the closest point of the eye and the light to the plane/line. For the purposes of this question, the dot at the front of the eye is used as the eye point, and (for the first two parts) the intensity of the light does NOT attenuate with distance.



1. Assume the plane/line only has a diffuse material component. Mark with a 'D' on the figure where the plane/line will appear brightest to the viewer.
2. Assume instead that the plane/line only has a specular material component. Mark with an 'S' on the figure where the plane/line will appear brightest to the viewer.
3. Suppose now the light has r^2 attenuation. How will the location of the brightest point change if the plane/line only has a diffuse material component?
4. Again suppose the light has r^2 attenuation. How will the location of the brightest point change if the plane/line only has a specular material component?

(Q6) Flat shading, Gouraud shading, and Phong shading are methods for shading rasterized polygons.

1. Describe the major difference between Gouraud shading and Phong shading
2. Describe a specific situation (or scene) where Gouraud shading and Phong shading produce substantially different results
3. Provide two good reasons why one might prefer flat shading to Phong shading.

(Q7) Assume that the viewer and the light source are both located at z = ∞ above a terrain with identical vertex normals. Assuming identical material and light colors for ambient, diffuse, and specular, no attenuation or spotlight factors, and Gouraud shading, which of the following illuminations will be rendered as identical: (a) Ambient-only, (b) Diffuse-only, (c) Specular only.

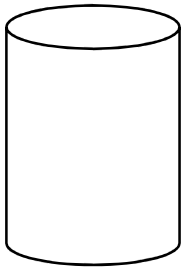
If we change the shading from Gouraud to Phong, keeping the other parameters the same, which of the above illuminations (a), (b), (c) will be rendered identically?

Ray Tracing

(Q8) Consider a ray e + td that hits a surface S at a point Q, where the normal to S at Q is n, and where S has only diffuse reflection material components (dr, dg, db). Further, assume there is a white point light at position L with RGB intensity values (I, I, I), and a white ambient light with RGB intensity values (A, A, A). In ray tracing, we want to know the intensity of light that travels back along this ray and eventually reaches the eye. For the situation described above, explain how we compute this intensity, noting any additional rays we cast and any local lighting computations we perform.

(Q9) Infinite cylinder along y of radius r axis has equation

$x^2 + z^2 - r^2 = 0$



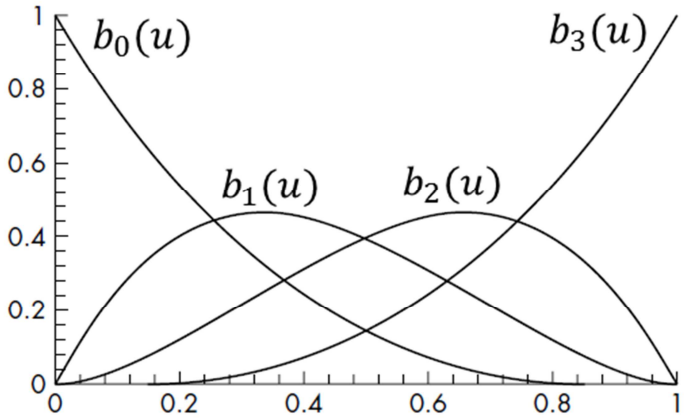
Derive an expression for the ray/cone intersection that gives the values of t at the intersections of the cone with the ray defined by the equation ***e + td***, where ***e*** is the ray origin and ***d*** is its direction.

(Q10) Consider the following ray/surface intersection function.

```
function IntersectX(Point p, Vector d)
t = (1 - (p.x + p.y + p.z)) / (d.x + d.y + d.z)
if (t > 0)
        return t;
else return PLUS_INFINITY;
```

With what surface does this function return the intersection?

Curves and Surfaces

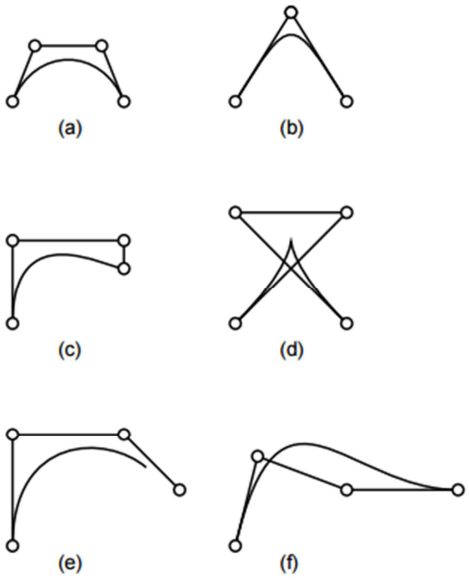


(Q11) Shown above is the graph of the four cubic basis functions for cubic Bezier curves. Given four control points $p_0,\ p_1, p_2, p_3$, a Bezier curve C(t) may be computed as a weighted combination of these control points, weighted by the basis functions evaluated at parameter t. Briefly explain how you may conclude the following three statements by examining the values in the graph:

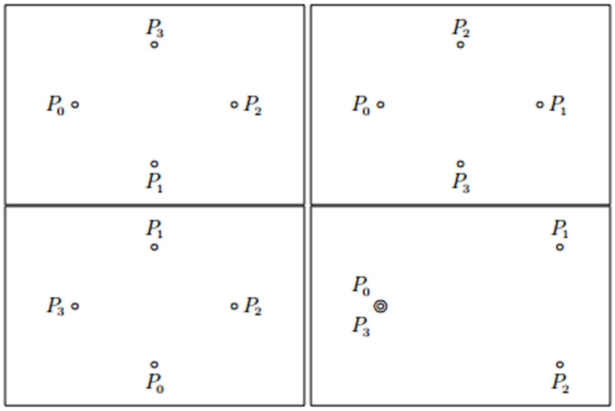
a. Bezier curves interpolate the first and last control points (p_0 and p_3)
b. Bezier curves do not exhibit local control – the entire curve moves when any of the four control points is moved.
c. Bezier curves are contained entirely within the convex hull of the control polygon.

(Q12) Explain why it would be impossible to construct a simple, closed cubic Bezier curve that has C^1 continuity everywhere. Include a small diagram if you want.

(Q13) Below are six curves and their "control points/polygon." Two of the control polygons are the Bezier control polygon for the curve drawn with it; the other four are not. Indicate which of the two control polygons are Bezier control polygons for the corresponding curve and which four are not Bezier control polygons for the corresponding curve. Justify your answer for the control polygons that are not Bezier control polygons. You may assume that none of the control points overlap or are repeated.

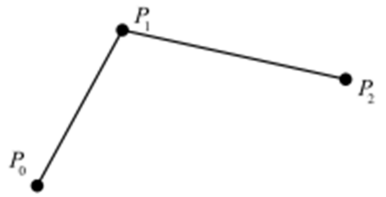


(Q14) The following pictures show possible arrangements of control points for a cubic Bezier curve. In the case of each picture, make as accurate a sketch of the curve as possible.



(Q15) The diagram below shows three control points P0, P1 and P2 that together define a quadratic B´ezier spline P(t), t ∈ [0, 1]. Use de Casteljau's algorithm to evaluate the spline at t = 1/3 .

a. Sketch the intermediate lines and points that are used in the algorithm and clearly label the point P (1/3)



b. 2. Based on the formula for an affine combination of two points, use de Casteljau's algorithm to derive an explicit equation for P(t) in terms of P0, P1 and P2.
c. Suppose that we wish to place three additional control points Q0, Q1 and Q2 defining a second B´ezier spline segment, parameterized over [1, 2]. Give formulas for whichever of the Qs are necessary to guarantee that the two curve segments will be joined with C 1 continuity.

Subdivision

(Q16) If a cube with quadrilateral faces is subdivided k times using Catmull-Clark subdivision, how many faces will the resulting shape have?

(Q17) What kinds of objects are generally suitable for modeling using subdivision surfaces? Give an example of where using subdivision surfaces would not be appropriate.

(Q18) Suppose we apply subdivision surfaces starting with a box shape. What happens to the resulting surface if we add additional vertices near one corner without altering the shape of the box

---

(Q19) A mesh is made up of two triangles, a hexagon (6 sides), and 100 quadrilaterals. All vertices in the mesh are connected to exactly 4 other vertices. The mesh is closed (it has no edges). (note: this mesh is hypothetical, I'm not sure it can actually exist, but that doesn't matter for this question). This mesh is used as the starting surface for Catmull-Clark Subdivision.

After the first subdivision, how many polygons will not be quadrilaterals?

After the first subdivision, how many vertices have less than 4 neighbors?

After the first subdivision, how many vertices have more than 4 neighbors?