Subdivision

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Subdivision curves and surfaces

- Subdivision surfaces are polygon mesh surfaces generated from a base mesh through an iterative process that smooths the mesh while increasing its density

- An alternative representation
- Avoid parameterizations
- Easy approach
- Fast computation
- Tender to generate smooth shape

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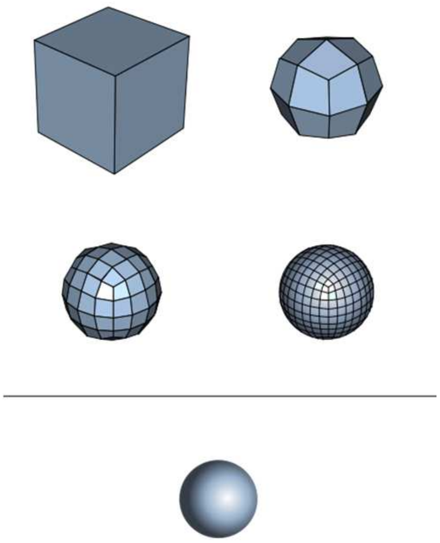
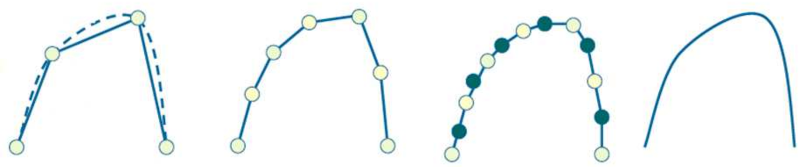
Subdivision curves and surfaces



A Catmull-Clark subdivision surface. Made by Romainbehar in K-3D

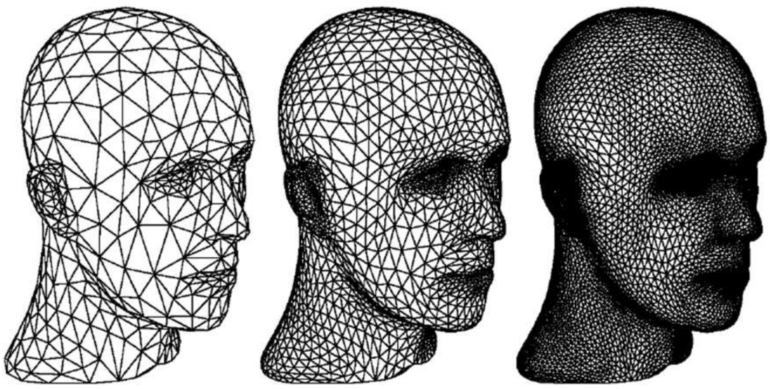
2D Curve

- Approach a smooth curve from rough starting shape
- Use iterative refinement process



Subdivision surface

- Use same strategy on 3D meshes



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Subdivision methods

- By interpolation
  - Surfaces/Curve will *pass through* starting data set

- By approximation
  - Surface/Curve will not necessarily pass through the starting data set
  - Get close to the starting data set

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Chaikin's Algorithm, 1974

- Start with a given set of control points P_i, perform a computation that results in a new set of points P_i^1 and repeat the process, producing more and more sets of points P_i^k

$$P_0, P_1, \dots, P_n$$
$$P_0^1, P_1^1, \dots, P_{n1}^1$$
$$P_0^2, P_1^2, \dots, P_{n2}^2$$
$$\dots$$
$$P_0^k, P_1^k, \dots, P_{nk}^k$$

- Each point P_j^k is computed as a weighted sum of the points P_i^{k-1} of the previous iteration

$$P_j^k = \sum_{i=0}^{n_k - 1} a_{ijk} P_i^{k-1} = [a_{0jk} \quad a_{1jk} \quad \cdots \quad a_{n_{k-1}jk}] \begin{bmatrix} P_0^{k-1} \\ P_1^{k-1} \\ \vdots \\ P_{n_k-1}^{k-1} \end{bmatrix}$$

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Chaikin's Algorithm, 1974

- Each iteration can be completely described by its coefficient matrix

$$\begin{bmatrix} P_0^k \\ P_1^k \\ \vdots \\ P_{n_k}^k \end{bmatrix} = \begin{bmatrix} a_{00k} & a_{10k} & \cdots & a_{n_{k-1}0k} \\ a_{01k} & a_{11k} & \cdots & a_{n_{k-1}1k} \\ \vdots & \vdots & & \vdots \\ a_{0n_kk} & a_{1n_kk} & \cdots & a_{n_{k-1}n_kk} \end{bmatrix} \begin{bmatrix} P_0^{k-1} \\ P_1^{k-1} \\ \vdots \\ P_{n_k-1}^{k-1} \end{bmatrix} = M_k \begin{bmatrix} P_0^{k-1} \\ P_1^{k-1} \\ \vdots \\ P_{n_k-1}^{k-1} \end{bmatrix}$$

- M_k has $n_k + 1$ rows and $n_{k-1} + 1$ columns

- Chakin's refinement: start with n + 1 points $P_0, P_1, \dots P_n$ and apply

$$P_{2j}^{k+1} = \frac{3}{4} P_j^k + \frac{1}{4} P_{j+1}^k, \qquad P_{2j+1}^{k+1} = \frac{1}{4} P_j^k + \frac{3}{4} P_{j+1}^k$$

- Each iteration doubles the number of points and brings the points closer to the curve

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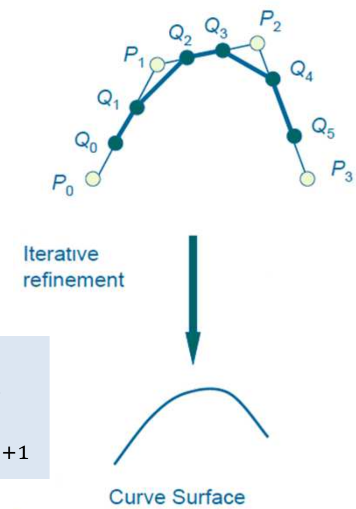
Chaikin's Algorithm, 1974

$$Q_0 = \frac{1}{4}P_0 + \frac{3}{4}P_1$$
$$Q_1 = \frac{3}{4}P_0 + \frac{1}{4}P_1$$

$$Q_2 = \frac{1}{4}P_1 + \frac{3}{4}P_2$$
$$Q_3 = \frac{3}{4}P_1 + \frac{1}{4}P_2$$

$$Q_4 = \frac{1}{4}P_2 + \frac{3}{4}P_3$$
$$Q_5 = \frac{3}{4}P_2 + \frac{1}{4}P_3$$

$$Q_{2i} = \frac{1}{4}P_i + \frac{3}{4}P_{i+1}$$
$$Q_{2i+1} = \frac{3}{4}P_i + \frac{1}{4}P_{i+1}$$

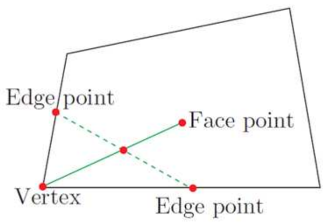


Doo-Sabin Surfaces, 1978

- Each new point P_{ij}^1 is a weighted sum of four points: a vertex point, two edge points, and a face point
- For example:

$$P_{00}^1 = \frac{1}{16}(9P_{00}^0 + 3P_{10}^0 + 3P_{01}^0 + 1P_{11}^0) = \frac{1}{4}(4V + E_1 + E_2 + F)$$

where V is the vertex point P_{00}^0, E_1 is the average of P_{00}^0 and P_{01}^0, E_2 is the average of P_{00}^0 and P_{10}^0, and F is the average of the four corners of the polygon being subdivided



Doo-Sabin Steps

- Consider face *F*
  - It now contains some new points P_i^1 Connect them so that they form a new polygon
  - This polygon will become a face in the new, refined surface
  - Repeat for all faces *F*

- Consider again a vertex P_i^0 on the original mesh. Such a vertex is normally common to several faces. For each of those faces
  - Find the new point that's nearest P_i^0 Connect those points to each other to form a new polygon
  - This polygon will also become a face in the new, subdivided surface
  - Repeat for all vertices

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Doo-Sabin Steps

- Consider an edge of the original mesh of points. There will normally be two faces adjacent to this edge and they will have new points P_i^1
  - Connect the new points around the edge to form a new polygon
  - This polygon will also become a face in the new, subdivided surface
  - Repeat this step for all edges

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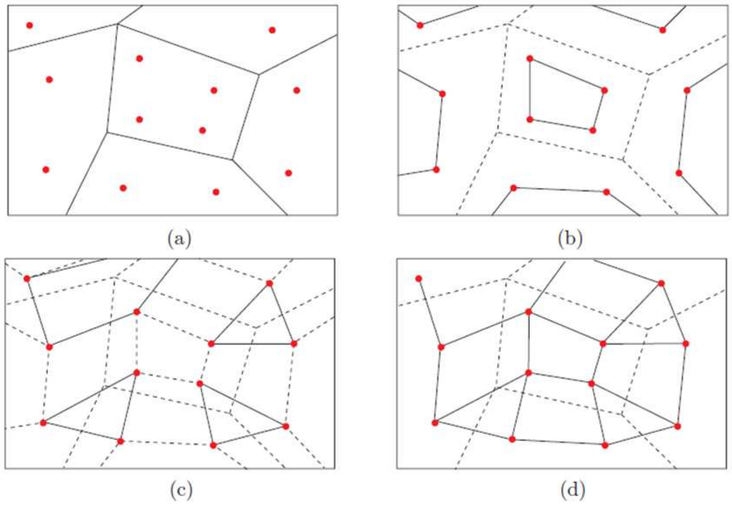
Doo-Sabin - Example



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Catmull-Clark Surfaces, 1978

- Generate new face, edge, and vertex points and connect them
- A face point is calculated for each face of the original mesh. The point is simply the average of all the points that bound the face
- An edge point is created for each interior edge of the polygonal surface. The point is the average of the midpoint of the edge and of the two face points on both sides of the edge.
- A vertex point is generated for each interior vertex **P** of the original mesh. The point is the average of **Q**, 2**R**, and **S**(n−3)/4, where **Q** is the average of the face points on all the faces adjacent to **P**, **R** is the average of the midpoints of all the edges incident on **P**, and **S** is simply **P** itself.

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Catmull-Clark Surfaces, 1978

- After the new points have been generated, they are connected according to the following rules:
- Each face point is connected to all the edge points of the interior edges bounding its face
- Each new vertex point is connected to all the edge points that were used in calculating it

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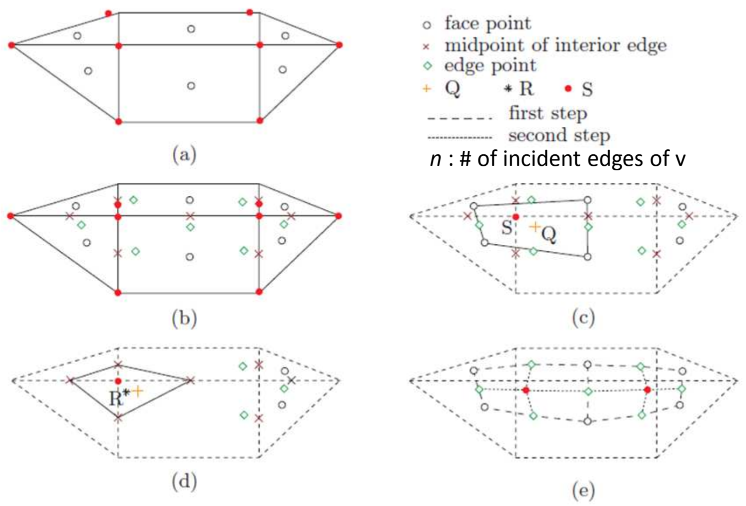
Catmull-Clark - Example



n : # of incident edges of v

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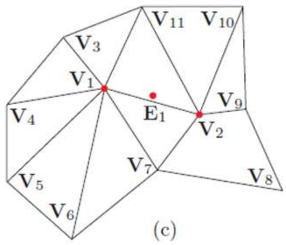
Loop Surface, 1987

- Loop's algorithm only works for triangle meshes
- Loop's algorithm computes a new edge point for each edge and a new vertex for each vertex
- Consider the edge between V_1 and V_2, the new edge point E_1 is constructed as $E_1 = \frac{3}{8}(V_1 + V_2) + \frac{1}{8}(V_{11} + V_7)$



(c)

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Loop Surface, 1987

- A new vertex point is also constructed as a weighted sum
- The new vertex for V_1, is computed as the sum $\frac{5}{8}V_1 + \frac{3}{8}Q_1$ where Q_1 is the average of the vertices of all the triangles sharing V_1

$$Q_1 = (V_3 + V_4 + V_5 + V_6 + V_7 + V_2 + V_{11})/7$$

- In general, a new vertex is computed as the weighted sum

$$\alpha_n V_i + (1 - \alpha_n)Q_i$$

Where $\alpha_n = (\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n})^2 + \frac{3}{8}$

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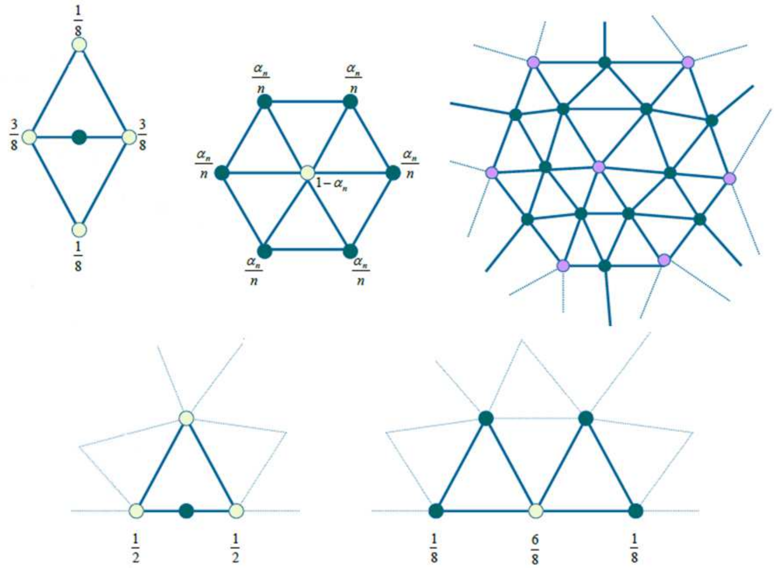
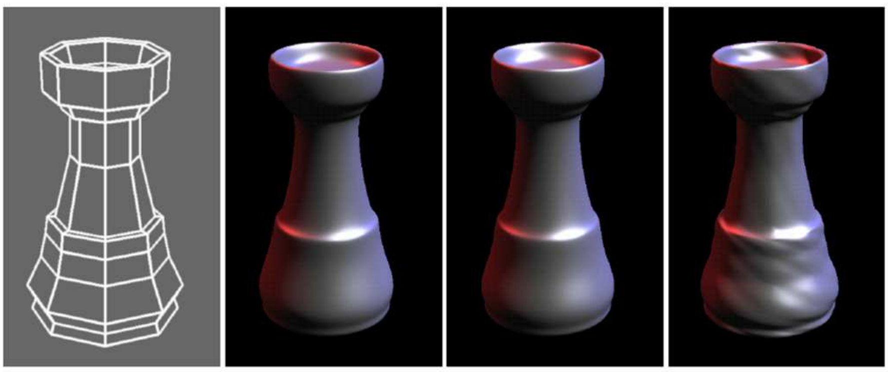
Loop Surface - Example



Different Results



Initial mesh *Loop* *Catmull-Clark* *Catmull-Clark,after triangulation*