Chapter 1
The Role of Algorithms in Computing

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Outlines: Introduction

- Algorithm: Definitions
- Algorithm: Characteristics & Components
- Algorithm: Examples
  - **Finding a *smallest* number among a sequence of *n* numbers**
- Algorithm: Efficiency

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Computational problems

- A computational problem specifies an input-output relationship
  - What does the input look like?
  - What should the output be for each input?
- Example:
  - Input: an integer number n
  - Output: Is the number prime?
- Example:
  - Input: A list of names of people
  - Output: The same list sorted alphabetically
- Example:
  - Input: A picture in digital format
  - Output: An English description of what the picture shows

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Algorithms

- A tool for solving a well-specified computational problem

- An algorithm is said to be correct if, for every input instance, it halts with the correct output
- Algorithms must be:
  - Correct: For each input produce an appropriate output
  - Efficient: run as quickly as possible, and use as little memory as possible – more about this later

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Problems and Algorithms

- We need to solve a computational problem
  - "Convert a weight in pounds to Kg"
- An algorithm specifies how to solve it, e.g.:
  1. Read weight-in-pounds
  2. Calculate weight-in-Kg = weight-in-pounds * 0.455
  3. Print weight-in-Kg
- A computer program is a computer-executable description of an algorithm

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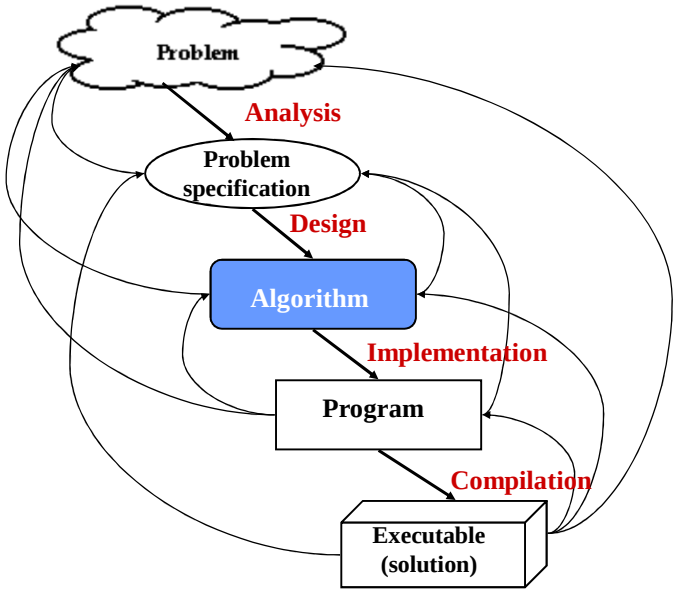
The Problem-solving Process



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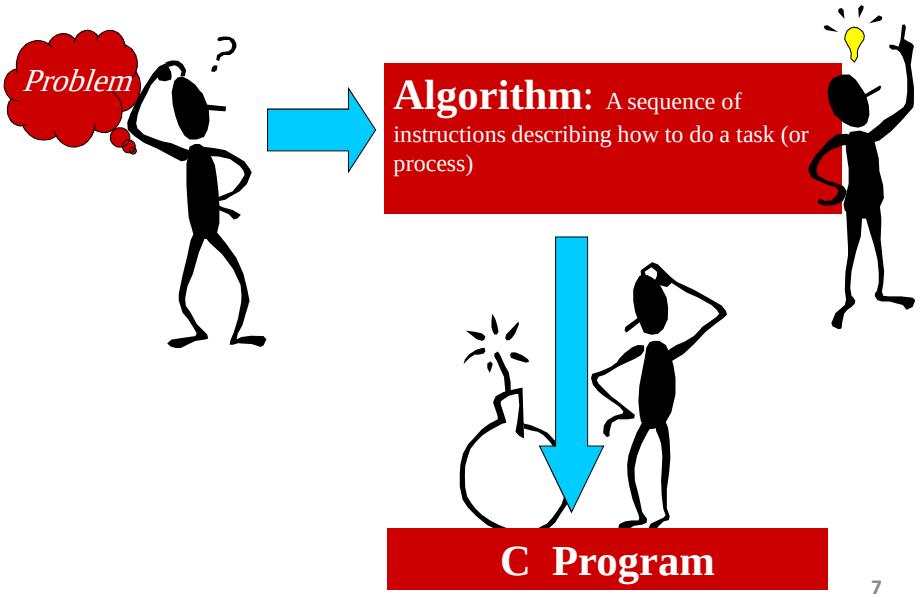
From Algorithms to Programs



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Components of an Algorithm

- Variables and values
- Instructions
- Sequences
- Procedures
- Selections
- Repetitions
- Documentation

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Values

- Represent quantities, amounts or measurements
- May be numerical or alphabetical (or other things)
- Often have a unit related to their purpose
- Example:

```
procedure Sum1_to_n(num)
{
        count = 1
        sum = 0
        while (count <= num)
        {
           add count to sum
           add 1 to count
        }
}
```

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Variables

Are <u>containers</u> for values – places to store values

Example:

```
procedure Sum1_to_n(num)
{
        count = 1
        sum = 0
        while (count <= num)
        {
           add count to sum
           add 1 to count
        }
}
```

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Components of an Algorithm

- **Values and Variables**
- Instruction
- Sequence (of instructions)
- Procedure (involving instructions)
- Selection (between instructions)
- Repetition (of instructions)
- Documentation (beside instructions)

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Instructions (Primitives)

- Some action that is simple...
- ...and unambiguous...
- ...that the system knows about...
- ...and should be able to actually do
- Examples:

```
procedure Sum1_to_n(num)
{
        count = 1
        sum = 0
        while (count <= num)
        {
           add count to sum
           add 1 to count
        }
}
```

Directions to perform specific actions on values and variables.

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Components of an Algorithm

- **Values and Variables**
- **Instruction**
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- Selection (between instructions)
- Repetition (of instructions)
- Documentation (beside instructions)

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Sequence

- A series of instructions
- ...to be carried out one after the other...
- ...without hesitation or question
- Example:
  - How to cook Gourmet Meal

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Sequence -- Example

1. Open freezer door
2. Take out Gourmet Meal™
3. Close freezer door
4. Open microwave door
5. Put Gourmet Meal™ on carousel
6. Shut microwave door
7. Set microwave on high for 5 minutes
8. Start microwave
9. Wait 5 minutes
10. Open microwave door
11. Remove Gourmet Meal™
12. Close microwave door

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Components of an Algorithm

- **Values and Variables**
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Procedure

- A named sequence of instructions
- So that you can
  - Refer to it collectively (by name)
  - ...instead of individually (by each instruction in the sequence)
- Example:
  - Drive_To_Uni

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Procedure -- Example

procedure Drive_To_Uni
{
1. find car keys
2. disable car alarm
3. open car door
4. get in car
5. shut car door
6. put keys in ignition
7. start car
8. back car out of driveway
9. drive to end of street
10. turn right
11. drive to end of street
12. turn left

...etc...etc...etc

...etc...etc...etc...

52. find parking space
53. pull into parking space
54. turn off engine
55. remove keys from ignition
56. open car door
57. get out
58. shut car door
59. lock car door
60. enable alarm

}

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Procedure – Example (cont)

```
procedure Do_Wednesday
{
  Wake_up
  Have_Shower
  Eat_Breakfast
  Drive_To_Uni
  Sit_1301_Lecture
  ...etc...etc...etc...
  Drive_From_Uni
  ...etc...etc...etc...
}
```

```
procedure Do_Week
{
  Do_Monday
  Do_Tuesday
  Do_Wednesday
  Do_Thursday
  ...etc...etc...etc...
}
```

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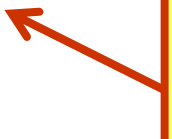
Procedure – Example (cont)

```
procedure Do_Wednesday
{
  Wake_up
  Have_Shower
  Eat_Breakfast
  Drive_To_Uni
  Sit_1301_Lecture
  ...etc...etc...etc...
  Drive_From_Uni
  ...etc...etc...etc...
}
```



In this subject, we also use the following words to refer to a "Procedure" :

Sub-routine
Module
Function

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Procedure – Example (cont)

```
procedure Do_Wednesday
{
  Wake_up
  Have_Shower
  Eat_Breakfast
  Drive_To_Uni
  Sit_1301_Lecture
  ...etc...etc...etc...
  Drive_From_Uni
  ...etc...etc...etc...
}
```

We use **brackets** to mark the beginning and end of a sequence.

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Procedure – Example (cont)

```
procedure Do_Wednesday
{
  Wake_up
  Have_Shower
  Eat_Breakfast
  Drive_To_Uni
  Sit_1301_Lecture
  ...etc...etc...etc...
  Drive_From_Uni
  ...etc...etc...etc...
}
```

An instruction invoking a procedure is known as a **"procedure call"**

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Procedure

- A procedure may have a set of parameters

```
procedure customerService ( myName ,timeOfDay )
{
  say "Good timeOfDay"
  say "My name is myName"
  say "How can I help you?"
}
```

```
customerService ( "Ann", "Morning")
customerService ("Ann", "Afternoon")
customerService ( "Jeff", "Afternoon")
```

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Components of an Algorithm

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Selection

- An instruction that decides which of two possible sequences is executed
- The decision is based on a single **true/false** condition
- Examples:
  - Car repair
  - Reciprocals

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Selection Example -- Car Repair

```
if (motor turns)
then
{
    CheckFuel
    CheckSparkPlugs
    CheckCarburettor
}
else
{
    CheckDistributor
    CheckIgnitionCoil
}
```

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Selection Example – Car Repair (cont)

```
if (motor turns)
then
{
    CheckFuel
    CheckSparkPlugs
    CheckCarburettor
}
else
{
    CheckDistributor
    CheckIgnitionCoil
}
```

Should be a true or false condition.

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Selection Example -- Car Repair (cont)

```
if (motor turns)
then
{
    CheckFuel
    CheckSparkPlugs
    CheckCarburettor
}
else
{
    CheckDistributor
    CheckIgnitionCoil
}
```

Sequence if the condition is true.

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Selection Example -- Car Repair (cont)

```
if (motor turns)
then
{
    CheckFuel
    CheckSparkPlugs
    CheckCarburettor
}
else
{
    CheckDistributor
    CheckIgnitionCoil
}
```

Sequence if the condition is false.

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Selection Example -- Reciprocals

Q. Give an algorithm for computing the reciprocal of a number.

Examples:

Reciprocal of 2: 1/2

Reciprocal of -3/4:
1/(-3/4) = -4/3

Reciprocal of 0:
"undefined"

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Selection Example – Reciprocals (cont)

Algorithm:

```
input Num
 if (Num is not equal 0)
 then
 {
        output 1/Num
 }
 else
 {
        output "infinity"
 }
```

Q. Give an algorithm for computing the reciprocal of a number.

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Selection Example-- Reciprocals

Algorithm:

```
input Num
 if (Num is not equal 0)
 then
 {
      output 1/Num
 }
 else
 {
      output "infinity"
 }
```

Num is a variable whose value depends on the actual number the user provides.

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Selection Example – Reciprocals (cont)

Algorithm:

```
input Num
if (Num is not equal 0)
then
{
      output 1/Num
}
else
{
      output "infinity"
}
```

Condition depends on the value of **Num**

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Selection Example – Reciprocals (cont)

Algorithm:

```
input Num
 if (Num is not equal 0)
then
{
      output 1/Num
}
else
{
      output "infinity"
}
```

For a given value of **Num**, only one of these two sequences can be executed

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Selection Example – Reciprocals (cont)

Algorithm:

```
input Num
if (Num is not equal 0)
then
{
    output 1/Num
}
else
{
    output "infinity"
}
```

Executed if **Num** is *not* equal to 0

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Selection Example – Reciprocals (cont)

Algorithm:

```
input Num
if (Num is not equal 0)
then
{
    output 1/Num
}
else
{
    output "infinity"
}
```

Executed if **Num** *is equal* to 0

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Selection -- Exercise

Will the following algorithms produce the same output?

Algorithm 1:

```
input Num
if (Num is not equal 0)
then
{
      output 1/Num
}
else
{
      output "infinity"
}
```

Algorithm 2:

```
input Num
if (Num is not equal 0)
then
{
      output 1/Num
}

output "infinity"
```

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Selection – Several Conditions

- What if several conditions need to be satisfied?

```
if ( today is Wednesday and the time is 10.00am )
 then
 {
      Go to CSE1301 Lecture
 }
 else
 {
      Go to Library
 }
```

Solution 1

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Selection – Several Conditions (cont)

```
if ( today is Wednesday )
then
{
    if ( the time is 10.00am )
    then
    {
        Go to CSE1301 Lecture
    }
}
else
...etc...etc...etc...
```

Often called a "**nested selection**"

Solution 2

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Selection – At Least One of Several Conditions

- What if at least one of several conditions needs to be satisfied?

```
if ( I feel hungry or the time is 1.00pm or my mate has his eye on
     my lunch )
then
{
      Eat my lunch now
}
```

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Repetition

- Repeat an instruction...
  - ...while (or maybe until) some true or false condition occurs
  - Test the condition each time before repeating the instruction
- Also known as iteration or loop
- Example:
  - Algorithm for finding the summation from 1 to a given number

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Repetition -- Example

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
      add 1 to count
   }
}
```

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Repetition – Example (cont)

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
      add 1 to count
   }
}
```

Condition is tested before sequence

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Repetition – Example (cont)

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
      add 1 to count
   }
}
```

Sequence may not get executed at all

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Repetition – Example (cont)

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
      add 1 to count
   }
}
```

Ensure initial values of variables used in the conditions are set correctly

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Repetition – Example (cont)

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
      add 1 to count
   }
}
```

Ensure the variables used in the conditions are updated in each iteration

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Repetition – Example (cont)

What if we don't increment count?

```
procedure Sum1_to_n(num)
{
   count = 1
   sum = 0
   while (count <= num)
   {
      add count to sum
   }
}
```

Infinite loop

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Components of an Algorithm

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Documentation

- Records what the algorithm does
- Describes how it does it
- Explains the purpose of each component of the algorithm
- Notes restrictions or expectations

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Documentation -- Example

```
// This procedure finds the summation from 1 to a given number
procedure Sum1_to_n(num)
{
    // do initialization
    count = 1
    sum = 0
    while (count <= num)
    {
        // add count to sum
        sum = sum + count
        // increment count by 1
        count = count + 1
    }
}
```

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Components of an Algorithm

- **Values and Variables**
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A Simple Algorithm

- INPUT: a sequence of n numbers,
  - T is an array of n elements
  - T[1], T[2], …, T[n]
- OUTPUT: the smallest number among them

```
min  = T[1]
for i = 2 to n do
{
   if T[i] < min
      min = T[i]
}
Output min
```

- Performance of this algorithm is a function of **n**

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Describing Algorithms

- Algorithms can be implemented in any programming language
- Usually we use "pseudo-code" to describe algorithms
- Testing whether input **n** is prime:

```
for j = 2 to n-1
{
  if mod(n, j) = 0
     output "n is non prime" and halt
}
output "n is prime"
```

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Algorithm Efficiency

- Consider two sort algorithms
  - **Insertion sort**
    - takes $\mathbf{c_1 n^2}$ to sort **n** items
    - where $\mathbf{c_1}$ is a constant that does not depends on **n**
    - it takes time roughly proportional to $\mathbf{n^2}$
  - Merge Sort
    - takes $\mathbf{c_2\ n\ lg(n)}$ to sort **n** items
    - where $\mathbf{c_2}$ is also a constant that does not depends on **n**
    - **lg(n)** stands for $\mathbf{log_2\ (n)}$
    - it takes time roughly proportional to **n lg(n)**
  - Insertion sort usually has a smaller constant factor than merge sort
    - so that, $c_1 < c_2$
  - Merge sort is faster than insertion sort for large input sizes

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Algorithm Efficiency

- Consider now:
  - A **faster** computer **A** running **insertion sort** against
  - A **slower** computer **B** running **merge sort**
  - Both must sort an array of **one million** numbers
- Suppose
  - Computer **A** execute **one billion** (10^9) instructions per second
  - Computer **B** execute **ten million** (10^7) instructions per second
  - So computer **A** is **100** times faster than computer **B**
- Assume that
  - $c_1 = 2$ and $c_2 = 50$

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Algorithm Efficiency

- To sort one million numbers
  - Computer A takes

$$\frac{2 \cdot (10^6)^2 \text{ instructions}}{10^9 \text{ instructions/second}} = 2000 \text{ seconds}$$

  - Computer B takes

$$\frac{50 \cdot 10^6 \cdot \lg(10^6) \text{ instructions}}{10^7 \text{ instructions/second}} \approx 100 \text{ seconds}$$

- By using algorithm whose running time grows more slowly, Computer B runs 20 times faster than Computer A
- **For ten million numbers**
  - **Insertion sort takes ≈ 2.3 days**
  - **Merge sort takes ≈ 20 minutes**

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