The University of Jordan
King Abdulla II School for Information Technology
Computer Science Department
Computer Graphics – Midterm Exam – Fall 2016

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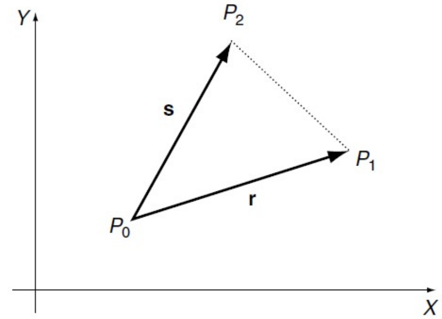
Q1 (4 points) Figure below shows three vertices of a triangle $P_0(x_0, y_0)$, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ formed in an anti-clockwise sequence. We can imagine that the triangle exists in 2D, therefore the z-coordinates are zero.

Given that the area of the triangle is $\frac{1}{2}|r \times s|$, show that the area can also be computed as:

$$area = \frac{1}{2}[(x_0y_1 - x_1y_0) + (x_1y_2 - x_2y_1) + (x_2y_0 - x_0y_2)]$$



$$r = P_1 - P_0 = \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ 0 \end{bmatrix}$$
$$s = P_2 - P_0 = \begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ 0 \end{bmatrix}$$

$$r \times s = \begin{bmatrix} i & j & k \\ x_1 - x_0 & y_1 - y_0 & 0 \\ x_2 - x_0 & y_2 - y_0 & 0 \end{bmatrix}$$

$$r \times s = \begin{bmatrix} 0 \\ 0 \\ (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0) \end{bmatrix}$$

$$|r \times s| = (x_1 - x_0)(y_2 - y_0) - (x_2 - x_0)(y_1 - y_0)$$
$$|r \times s| = x_1(y_2 - y_0) - x_0(y_2 - y_0) - x_2(y_1 - y_0) + x_0(y_1 - y_0)$$
$$|r \times s| = x_1y_2 - x_1y_0 - x_0y_2 - x_0y_0 - x_2y_1 - x_2y_0 + x_0y_1 + x_0y_0$$
$$|r \times s| = x_1y_2 - x_1y_0 - x_0y_2 - x_2y_1 - x_2y_0 - x_0y_1$$
$$|r \times s| = (x_0y_1 - x_1y_0) + (x_1y_2 - x_2y_1) + (x_2y_0 - x_0y_2)$$

$$area = \frac{1}{2}[(x_0y_1 - x_1y_0) + (x_1y_2 - x_2y_1) + (x_2y_0 - x_0y_2)]$$

Q2 (11 points) Answer each of the following questions

(a) **(3 points)** Given a circle centered at (0, 0) of radius r = 8 and P_0 = -7. Using Midpoint circle algorithm compute (x_0, y_0), (x_1, y_1) and P_1.

$(x_0, y_0) = (0, 8)$
$(x_1, y_1) = (1, 8)$
$P_1 = P_0 + 2x_{k+1} + 1 = -7 + 2\ (1) + 1 = -4$

(b) **(2 points)** Given the line with endpoints (40, 50) and (20, 0), find the line equation:

y = mx + b
$$m = \frac{\Delta x}{\Delta y} = \frac{0-50}{20-4} = \frac{-50}{-20} = 2.5$$
b = y – mx
b = 0 – (2.5) * 20 = -50
y = 2.5x - 50

(c) **(2 points)** For any given shape centered at ***(cx, cy)***, using rotation and translation only, provide the transformation matrix ***M*** that reflects the shape around the x-axis

$$M = \begin{bmatrix} 1 & 0 & cx \\ 0 & 1 & cy \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} cos(180) & -sin(180) & 0 \\ sin(180) & cos(180) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & -cx \\ 0 & 1 & -cy \\ 0 & 0 & 1 \end{bmatrix}$$

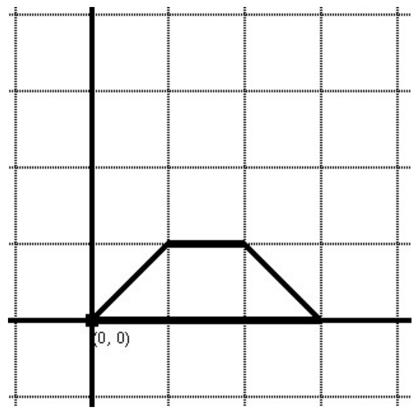
(d) **(2 points)** What is the approximate size, in Kilobytes, of a frame buffer that is needed to store 500×500 colored image

Frame buffer = 500 x 500 x 3 = 750kb

(e) **(2 points)** Express the homogeneous 2D transformation matrix $\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ as a rotation followed by a translation

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

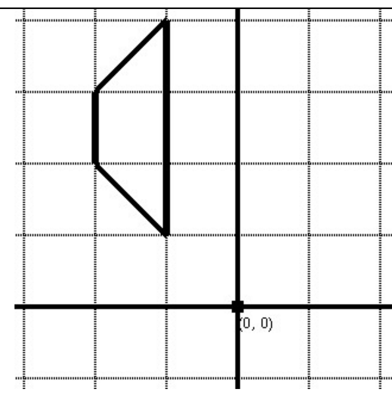
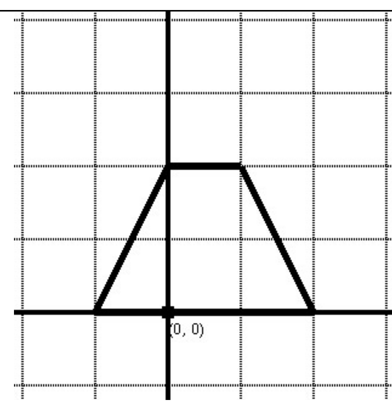
Q3 (10 points, 2 points each) Given the figure below that shows a shape in its initial position

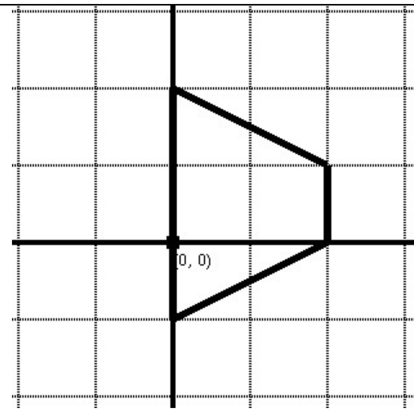


and the transformations:

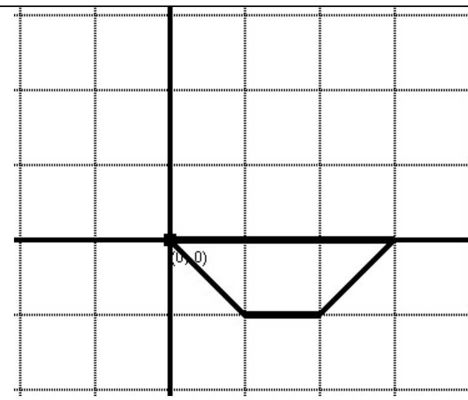
$T(t_x, t_y)$ ➔ Translation matrix
$S(s_x, s_y)$ ➔ Scaling matrix
$R(\theta)$ ➔ Rotation matrix

(a) Draw the shapes that result from applying the following transformations to the initial shape above.

R(90) . T(1, 1)	
S(1, 2) . T(-1, 0)	

S (2, 1).T (0, 2).R(270)	

(b) **Write down the matrix M (in symbolic form) to perform the transformation required to generate the figures below.**

T (3, 0).R (180)	
T (2, 2).R(180).S (4/3, 2)