# Chapter 3: Graphics Output Primitives

- Primitives: functions in graphics package that we use to describe picture element
- Points and straight lines are the simplest primitives
- Some packages include circles, conic sections, quadric surfaces, spline curves and surfaces, and polygon color area

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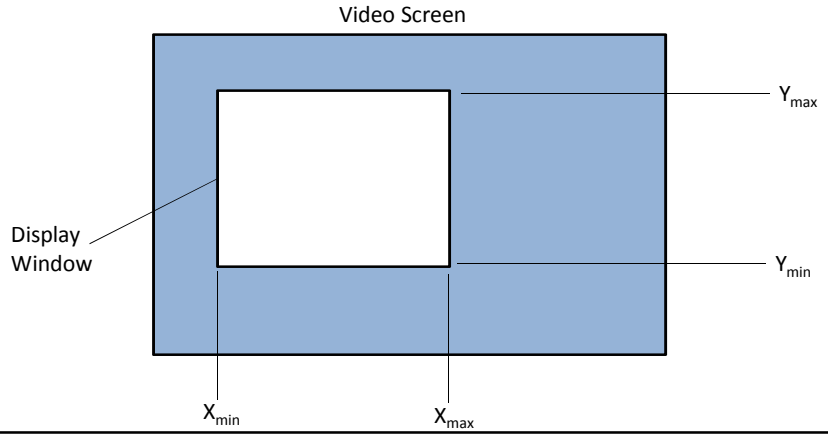
# Two-Dimensional World Coordinate Reference Frame in OpenGL

```
glMatrixMode (GL_PROJECTION);
glLoadIdentity ();
gluOrtho2D (xmin, xmax, ymin, ymax);
```

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# OpenGL Point Functions

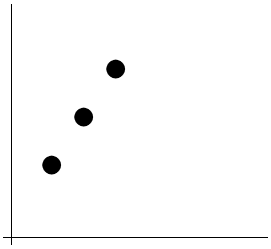
- OpenGL function to state coordinates values for point
- glVertex* ( ); where (*) suffix code required
- A glVertex must be placed between glBegin and glEnd

```
glBegin (GL_POINTS);
    glVertex* ( );
glEnd ();
```

```
glBegin (GL_POINTS);
    glVertex2i (50, 100);
    glVertex2i (75, 150);
    glVertex2i (100, 200);
glEnd ();
```

```
int pnt1 [] = {50, 100};
int pnt2 [] = {75, 150 };
int pnt13[] = {100, 200 };

glBegin (GL_POINTS);
    glVertex2iv (pnt1);
    glVertex2iv (pnt2);
    glVertex2iv (pnt3);
glEnd ();
```



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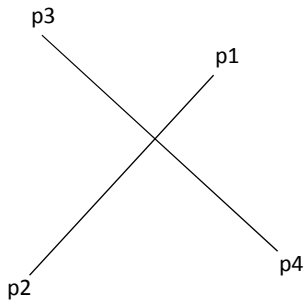
# OpenGL Line Functions

- Straight line segments between each successive endpoints

```
glBegin (GL_LINES);
    glVertex2iv (p1 );
    glVertex2iv (p2 );
    glVertex2iv (p3 );
    glVertex2iv (p4 );
glEnd ();
```

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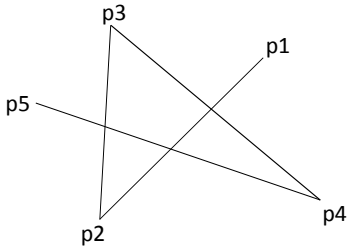
# OpenGL Line Functions

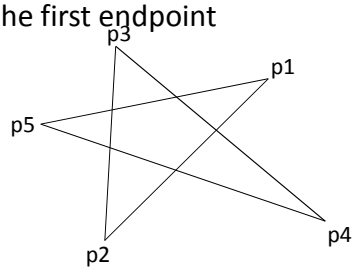
- A sequence of connected line segments between first and last endpoints

```
glBegin (GL_LINE_STRIP);
    glVertex2iv (p1 );
    glVertex2iv (p2 );
    glVertex2iv (p3 );
    glVertex2iv (p4 );
    glVertex2iv (p5);
glEnd ();
```




- Last coordinate is connected to the first endpoint

```
glBegin (GL_LINE_LOOP);
    glVertex2iv (p1 );
    glVertex2iv (p2 );
    glVertex2iv (p3 );
    glVertex2iv (p4 );
    glVertex2iv (p5);
glEnd ();
```

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# Line Drawing Algorithms

- Programmer specifies (x, y) values of end pixels
- Need algorithm to figure out which intermediate pixels are on line path
- Pixel (x, y) values constrained to integer values
- Actual computed intermediate line values may be floats
- Rounding may be required. E.g. computed point (10.48, 20.51) rounded to (10, 21)
- Rounded pixel value is off actual line path (jaggy!!)
- Sloped lines end up having jaggies
- Vertical, horizontal lines, no jaggies

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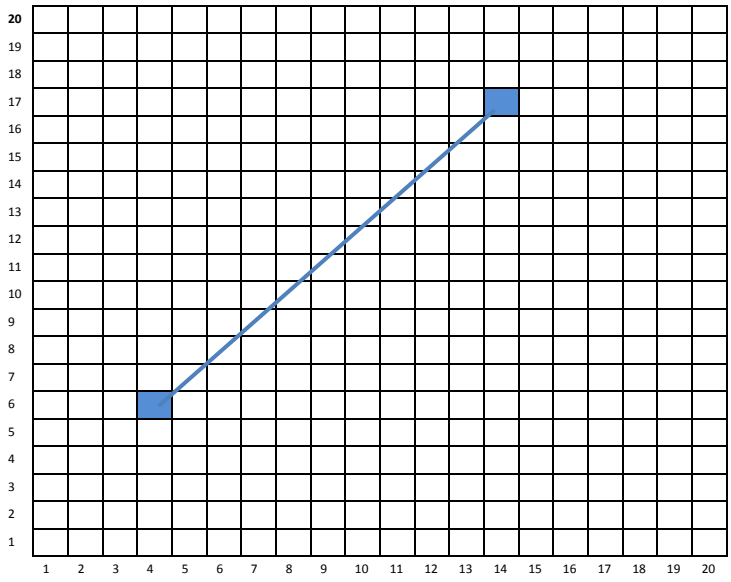
# Line Drawing Algorithms




Line:
(4, 6) → (14, 17)

Which intermediate pixels to turn on?

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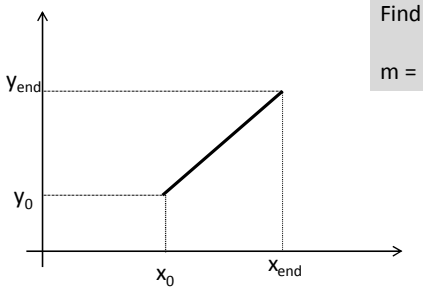
# Line Drawing Algorithms

- Slope-intercept line equation
  - $y = m \cdot x + b$
  - m : slope of the line, b : y intercept
  - Given two end points $(x_0, y_0)$ and $(x_{end}, y_{end})$, how to compute m and b?
  - $m = (y_{end} - y_0) / (x_{end} - x_0)$
  - $b = y_0 - m \cdot x_0$




Example:
Find the slope of (23, 41) and (125, 96)

m = (96 - 41) / (125 – 23) = 55 / 102 = 0.5392

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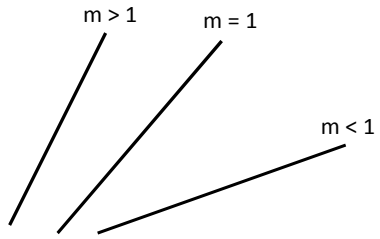
# DDA (Digital Differential Analyzer)

- A line is sampled at unit intervals in one coordinates and the corresponding integer values nearest the line path are determined for the other coordinate
- Case (m <= 1) : sample at unit x intervals ($\Delta x = 1$), computer successive y values as $y_{k+1} = y_k + m$
- Case (m > 1) : sample at unit y intervals ($\Delta y = 1$), computer successive x values as $x_{k+1} = x_k + 1 / m$

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# DDA (Digital Differential Analyzer)

$m < 1$, $y_{k+1} = y_k + m$

$x_1 = x_0 + 1$ $\quad y_1 = y_0 + m$

$x_2 = x_1 + 1$ $\quad y_2 = y_1 + m$

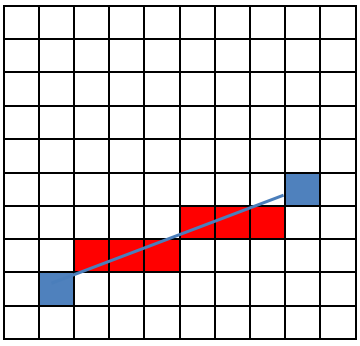
…

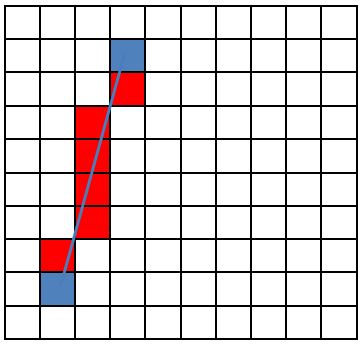
$m > 1$, $x_{k+1} = x_k + 1 / m$

$y_1 = y_0 + 1$ $\quad x_1 = x_0 + 1 / m$

$y_2 = y_1 + 1$ $\quad x_2 = x_1 + 1 / m$

…





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# DDA code

```
inline int round (const float a)
{
        return int (a + 0.5);
}

void lineDDA (int x0, int y0, int xEnd, int yEnd)
{
        int dx = xEnd - x0, dy = yEnd - y0, steps;
        float xIncrement, yIncrement, x = x0, y = y0;

        if (fabs (dx) > fabs (dy))
                steps = fabs (dx);
        else
                steps = fabs (dy);

        xIncrement = float (dx) / float (steps);
        yIncrement = float (dy) / float (steps);

        setPixel (round (x), round (y));

        for (int i = 0; i < steps; i++) {
                x += xIncrement;
                y += yIncrement;
                setPixel (round (x), round (y));
        }
}
```

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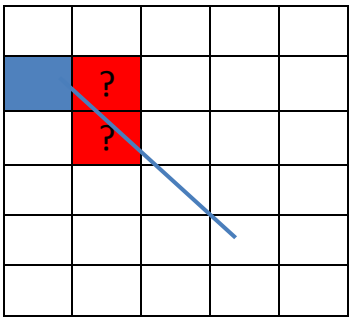
# Bresenham's Line Algorithm

- Accurate, efficient, and uses only incremental integer calculations
- Sampling at unit intervals, decide which of two possible pixel positions is closer to the line path

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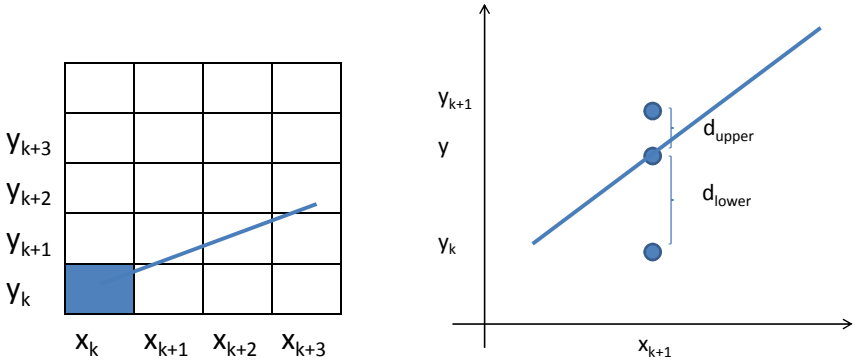
# Bresenham's Line Algorithm

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# Bresenham's Line Algorithm

- Consider positive slope less than 1.0
- Sampling at position $x_{k+1} = x_k + 1$, decide $(x_k + 1, y_k)$ or $(x_k + 1, y_k + 1)$

$$y = m\,(x_k + 1) + b$$

$$d_{lower} = y - y_k = m\,(x_k + 1) + b - y_k$$

$$d_{upper} = (y_k + 1) - y = y_k + 1 - m\,(x_k + 1) - b$$

Determine which pixel is closer based on the difference

$$d_{lower} - d_{upper} = 2m\,(x_k + 1) - 2y_k + 2b - 1$$

Substitute $m = \Delta y / \Delta x$ so it involves only integer calculations

$$d_{lower} - d_{upper} = 2(\Delta y/\Delta x)\,(x_k + 1) - 2y_k + 2b - 1$$

$$\Delta x\,(d_{lower} - d_{upper}) = 2\Delta y\,(x_k + 1) - \Delta x\,2y_k + \Delta x\,(2b - 1)$$

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## Bresenham's Line Algorithm

$P_k$ is a decision parameter for the kth step

$$P_k = \Delta x\,(d_{lower} - d_{upper})$$

$$P_k = 2\Delta y \,.\, x_k - 2\Delta x \,.\, y_k + \boxed{2\Delta y + \Delta x(2b - 1)} \rightarrow c$$

if $P_k$ is negative then pixel $y_k$ is closer that $(y_k + 1)$

At step k + 1

$$P_{k+1} = 2\Delta y \,.\, x_{k+1} - 2\Delta x \,.\, y_{k+1} + c$$

$$P_{k+1} - P_k = 2\Delta y\,(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

But $x_{k+1} = x_k + 1$

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x(y_{k+1} - y_k)$$

$y_{k+1} - y_k$ is either 0 or 1, depending on the sign of $P_k$

At the starting pixel position $(x_0, y_0)$ $\quad P_0 = 2\Delta y - \Delta x$

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## Bresenham's Line Algorithm for |m| < 1.0

1. Calculate the constants Δx, Δy, 2Δy, and 2Δy - 2Δx, and obtain the starting value for decision parameter as

$$P_0 = 2\Delta y - \Delta x$$

2. At each $x_k$ along the line, starting at k = 0, perform the following test. If $P_k < 0$, the next point to plot is $(x_k + 1, y_k)$ and

$$P_{k+1} = P_k + 2\Delta y$$

Otherwise, the next point is $(x_k + 1, y_k + 1)$ and

$$P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

3. Perform step 2 $\Delta x - 1$ times

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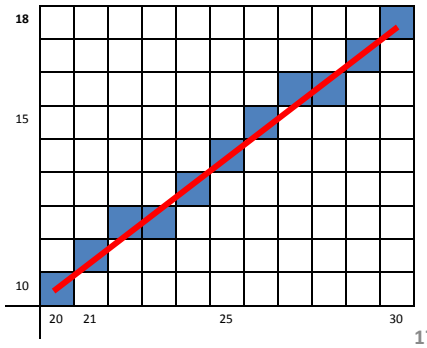
# Bresenham's Example

- Line with endpoints (20, 10) and (30, 18)
  - m = 0.8, $\Delta x = 10$, $\Delta y = 8$, $P_0 = 2\Delta y - \Delta x = 6$, $2\Delta y = 16$, $2\Delta y - 2\Delta x = -4$
- Plot initial point $(x_0, y_0) = (20, 10)$, and determine successive pixel positions

| k | $P_k$ | $(x_{k+1}, y_{k+1})$ |
|---|---|---|
| 0 | 6 | (21, 11) |
| 1 | 2 | (22, 12) |
| 2 | -2 | (23, 12) |
| 3 | 14 | (24, 13) |
| 4 | 10 | (25, 14) |
| 5 | 6 | (26, 15) |
| 6 | 2 | (27, 16) |
| 7 | -2 | (28, 16) |
| 8 | 14 | (29, 17) |
| 9 | 10 | (30, 18) |

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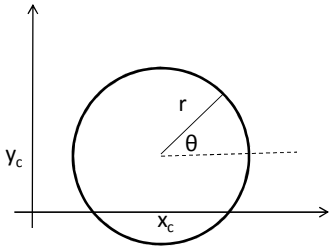
# Circle Drawing Algorithms

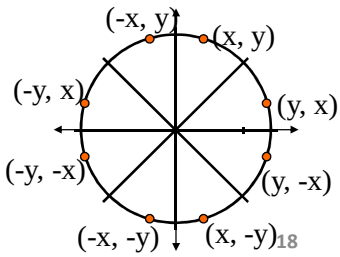
- A circle is a set of points that are a given distance r from center point $(x_c, y_c)$

$$(x - x_c)^2 + (y - y_c)^2 = r^2$$

$$y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$$




- The calculations are not very efficient.
  - The square (multiply) operations
- To make our circle drawing algorithm more efficient is that circles centred at (0, 0) have eight-way symmetry

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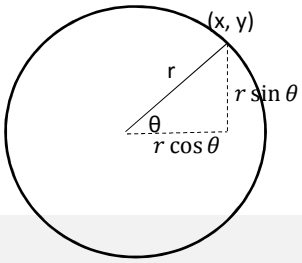
# Circle Drawing Algorithms

- Another way: use polar coordinates r and θ to express circle in parameter polar form

$$x = x_c + r\cos\theta$$
$$y = y_c + r\sin\theta$$

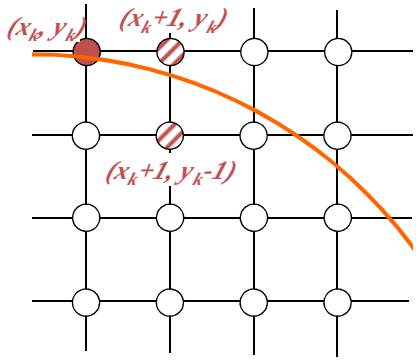



```
const float DEG2RAD = 3.14159/180;

void drawCircle (float radius)
{
   glBegin (GL_LINE_LOOP);

   for (int i=0; i < 360; i++) {
      float degInRad = i * DEG2RAD;
      glVertex2f (cos (degInRad) * radius, sin (degInRad) * radius);
   }
   glEnd();
}
```



# Midpoint Circle Algorithm

- Assume that we have just plotted point $(x_k, y_k)$
- The next point is a choice between $(x_k+1, y_k)$ and $(x_k+1, y_k-1)$
- We would like to choose the point that is nearest to the actual circle
- So how do we make this choice?



## Midpoint Circle Algorithm

- Let's rearrange the equation of the circle slightly to give us:

$$f_{circ}(x, y) = x^2 + y^2 - r^2$$

- The equation evaluates as follows:

$$f_{circ}(x, y) \begin{cases} < 0, \text{ if } (x, y) \text{ is inside the circle boundary} \\ = 0, \text{ if } (x, y) \text{ is on the circle boundary} \\ > 0, \text{ if } (x, y) \text{ is outside the circle boundary} \end{cases}$$

- By evaluating this function at the midpoint between the candidate pixels we can make our decision

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## Midpoint Circle Algorithm

- Assuming we have just plotted the pixel at $(x_k, y_k)$ so we need to choose between $(x_k + 1, y_k)$ and $(x_k + 1, y_k - 1)$
- Our decision variable can be defined as:

$$p_k = f_{circ}(x_k + 1, y_k - \tfrac{1}{2})$$
$$= (x_k + 1)^2 + (y_k - \tfrac{1}{2})^2 - r^2$$

- If $p_k < 0$ the midpoint is inside the circle and the pixel at $y_k$ is closer to the circle
- Otherwise the midpoint is outside and $y_k - 1$ is closer

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## Midpoint Circle Algorithm

- To ensure things are as efficient as possible we can do all of our calculations incrementally
- First consider:

$$p_{k+1} = f_{circ}\left(x_{k+1}+1, y_{k+1}-\tfrac{1}{2}\right)$$

$$= [(x_k+1)+1]^2 + \left(y_{k+1}-\tfrac{1}{2}\right)^2 - r^2$$

- or:

$$p_{k+1} = p_k + 2(x_k+1) + (y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k) + 1$$

- where $y_{k+1}$ is either $y_k$ or $y_k - 1$ depending on the sign of $p_k$



## Midpoint Circle Algorithm

- The first decision variable at position ($x_0$, $y_0$) = (0, r) is given as:

$$p_0 = f_{circ}(1, r-\tfrac{1}{2})$$

$$= 1 + (r-\tfrac{1}{2})^2 - r^2$$

$$= \tfrac{5}{4} - r$$

- Then if $p_k < 0$ then the next decision variable is given as:

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

- If $p_k > 0$ then the decision variable is:

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

## The Midpoint Circle Algorithm

1. Input radius $r$ and circle centre $(x_c, y_c)$, then set the coordinates for the first point on the circumference of a circle centred on the origin as:

$$(x_0, y_0) = (0, r)$$

2. Calculate the initial value of the decision parameter as:

$$p_0 = 5/4 - r$$

3. Starting with $k = 0$ at each position $x_k$, perform the following test. If $p_k < 0$, the next point along the circle centred on $(0, 0)$ is $(x_k+1, y_k)$ and:

$$p_{k+1} = p_k + 2x_{k+1} + 1$$

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## The Midpoint Circle Algorithm

Otherwise the next point along the circle is $(x_k+1, y_k-1)$ and:

$$p_{k+1} = p_k + 2x_{k+1} + 1 - 2y_{k+1}$$

4. Determine symmetry points in the other seven octants
5. Move each calculated pixel position $(x, y)$ onto the circular path centred at $(x_c, y_c)$ to plot the coordinate values:

$$x = x + x_c \qquad y = y + y_c$$

6. Repeat steps 3 to 5 until $x >= y$

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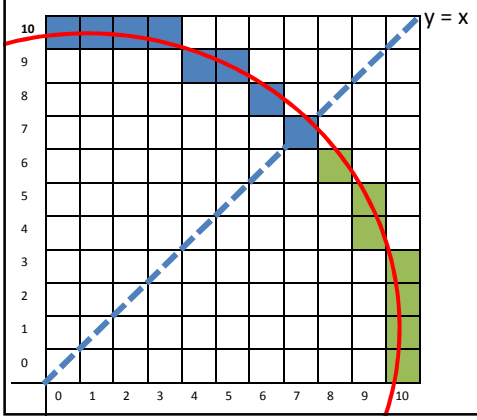
# Midpoint Circle Example

- Given radius r = 10
- Determine position along the circle octant in the first quadrant from x = 0 to x = y
  - $P_0 = 1 - r = -9$, $2x_0 = 0$, $2y_0 = 20$




| k | $P_k$ | $(x_{k+1}, y_{k+1})$ | $2x_{k+1}$ | $2y_{k+1}$ |
|---|---|---|---|---|
| 0 | -9 | (1, 10) | 2 | 20 |
| 1 | -6 | (2, 10) | 4 | 20 |
| 2 | -1 | (3, 10) | 6 | 20 |
| 3 | 6 | (4, 9) | 8 | 18 |
| 4 | -3 | (5, 9) | 10 | 18 |
| 5 | 8 | (6, 8) | 12 | 16 |
| 6 | 7 | (7, 7) | 14 | 14 |

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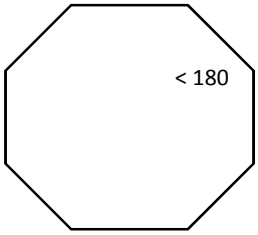
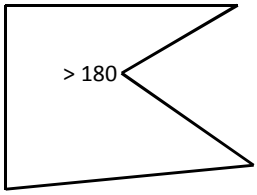
# Polygon

- Polygon is a plane figure specified by set of three or more coordinate positions, called vertices
- If all interior angles of a polygon are less than or equal to 180 degrees, the polygon is **convex**
- A polygon that is not convex is called a **concave** polygon




Convex




Concave

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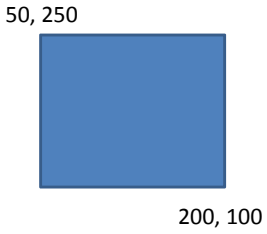
## OpenGL Polygon Fill Area Functions

- glRect* (x1, y1, x2, y2); one corner is at coordinate position (x1, y1) and the opposite corner is at position (x2, y2)
- glRect (200, 100, 50, 250);

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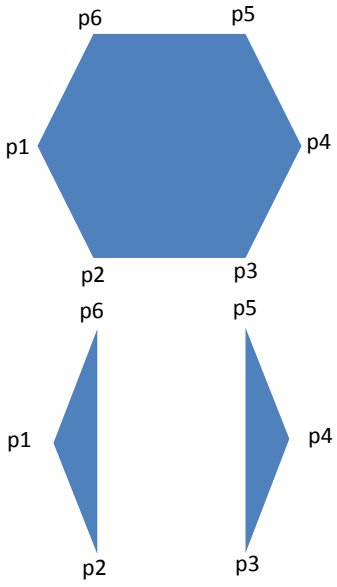
## OpenGL Polygon Fill Area Functions

```
glBegin (GL_POLYGON);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p3);
        glVertex2iv (p4);
        glVertex2iv (p5);
        glVertex2iv (p6);
glEnd ();

glBegin (GL_TRIANGLES);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p6);
        glVertex2iv (p3);
        glVertex2iv (p4);
        glVertex2iv (p5);
glEnd ();
```

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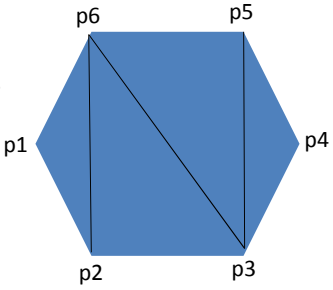
# OpenGL Polygon Fill Area Functions

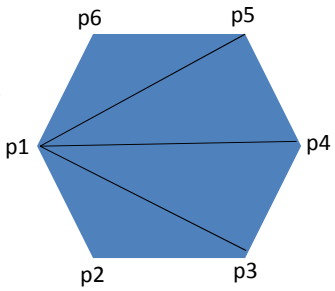
```
glBegin (GL_TRIANGLE_STRIP);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p6);
        glVertex2iv (p3);
        glVertex2iv (p5);
        glVertex2iv (p4);
glEnd ();
```

N – 2 triangles




```
glBegin (GL_TRIANGLE_FAN);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p3);
        glVertex2iv (p4);
        glVertex2iv (p5);
        glVertex2iv (p6);
glEnd ();
```

N – 2 triangles

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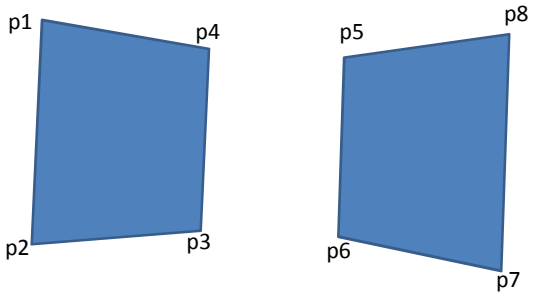
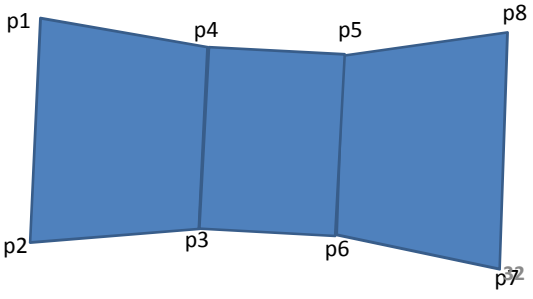
# OpenGL Polygon Fill Area Functions

```
glBegin (GL_QUADS);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p3);
        glVertex2iv (p4);
        glVertex2iv (p5);
        glVertex2iv (p6);
        glVertex2iv (p7);
        glVertex2iv (p8);
glEnd ();
```




```
glBegin (GL_QUAD_STRIP);
        glVertex2iv (p1);
        glVertex2iv (p2);
        glVertex2iv (p3);
        glVertex2iv (p4);
        glVertex2iv (p5);
        glVertex2iv (p6);
        glVertex2iv (p7);
        glVertex2iv (p8);
glEnd ();
```

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