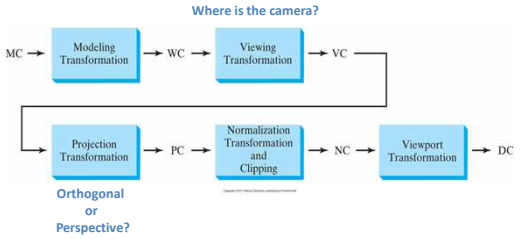
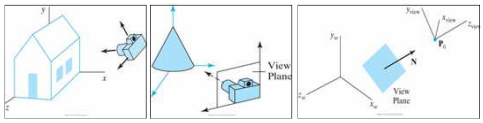
3D Viewing Pipeline



Viewing Coordinates

- Choose where to place a camera
- Set camera at position $P_0 = (x_0, y_0, z_0)$ which is called view point, eye position or camera position
- Specify a **view up vector V**, which defines the y_{view} direction
- Specify a **vector N** to be in direction from a reference point to P_0
- View plane (projection plane) is normally perpendicular to z_{view}



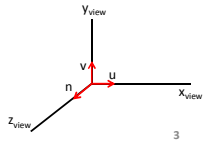
Transformation form World to Viewing

1. Translate the viewing-coordinate origin to world coordinate origin
2. Rotate to align x_{view}, y_{view}, and z_{view} axes with world x_w, y_w, and z_w

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R.T = \begin{bmatrix} u_x & u_y & u_z & -u.P_0 \\ v_x & v_y & v_z & -v.P_0 \\ n_x & n_y & n_z & -n.P_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$-u.P_0 = -x_0u_x - y_0u_y - z_0u_z$$
$$-v.P_0 = -x_0v_x - y_0v_y - z_0v_z$$
$$-n.P_0 = -x_0n_x - y_0n_y - z_0n_z$$



Projections

- Projection
  - Transformation from *n*-D coordinate system to *m*-D coordinate system, where $m < n$
- Our concern
  - $n = 3$ and $m = 2$
    $\Rightarrow$ projection from 3D to 2D
- Terminology
  - Projectors: Straight projection rays
  - Center of projection: Where the projectors emanated from
  - Projection plane: Where the projection forms

Projections

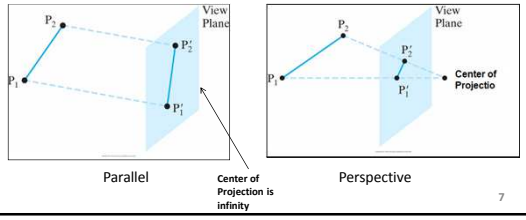
- Projection from 3D to 2D defined by
  - Projectors emanate from COP, pass though each point of the object, and intersect the projection plane
- Planar geometric projections
  - Projection is onto a plane
  - Referred to as "projections" here

Projections

- Two Basic Classes
  - Perspective
  - Parallel (Orthographic)
- Perspective
  - Distance between projection plane and COP is finite
  - Visual effect similar to human visual system
    - Perspective foreshortening:
      distance from COP longer, size smaller
  - Exact shape, measurement, parallelism not reserved

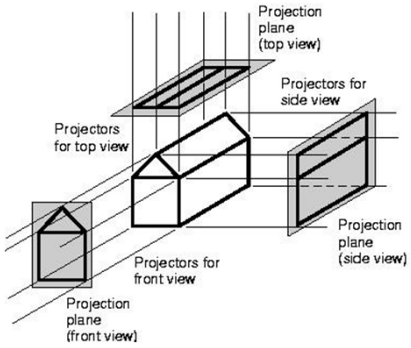
Projections

- Parallel
  - Distance between projection plane and COP is infinite
  - Less realistic view
    - No foreshortening
  - Exact measurement and parallelism preserved



7

Orthographic Parallel Projections



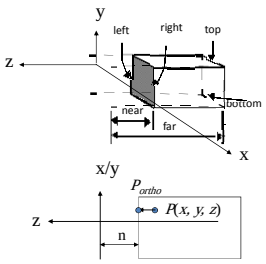
8

Orthographic Projection Math

- Projection parallel to z axis and perpendicular to projection plane

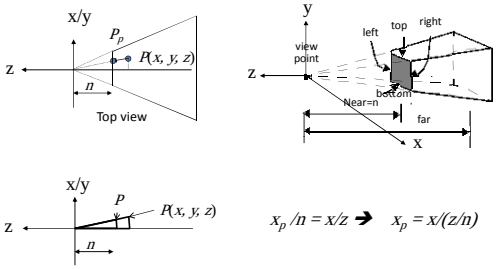
$$\begin{bmatrix} x \\ y \\ n \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & n \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$P_{ortho}(x, y, n)$



9

Perspective Projection Math



$x_p / n = x/z$ ➔ $x_p = x/(z/n)$

The projection onto the near clipping plane:

$$(x_p, y_p, z_p) = (\frac{x}{z/n}, \frac{y}{z/n}, n)$$

10

In homogeneous coordinates

In matrix form, perspective projection matrix:

$$M_{per} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{n} & 0 \end{bmatrix}$$

Transforming in homogeneous coordinates yields the general homogeneous point:

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{n} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = M_{per} P;$$

$$where \Rightarrow w = z / n$$

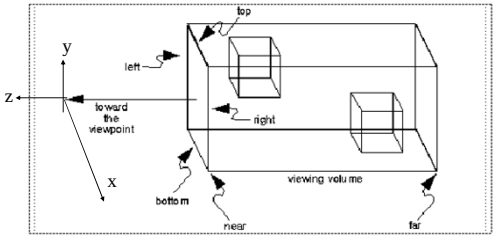
The 3D coordinates can be calculated from the homogeneous coordinates (perspective division, or divide by w):

$$(x_p, y_p, z_p) = (\frac{x}{w}, \frac{y}{w}, \frac{z}{w}) = (\frac{x}{z/n}, \frac{y}{z/n}, n)$$

11

OpenGL Orthographic Projection

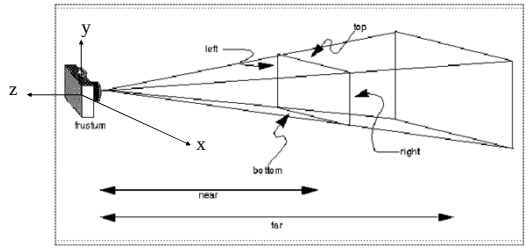
- void glOrtho(GLdouble left, GLdouble right, GLdouble bottom, GLdouble top, GLdouble near, GLdouble far);



12

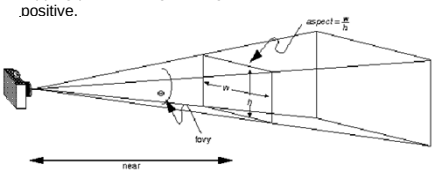
OpenGL Perspective Projection

- void glFrustum(GLdouble left, GLdouble right, GLdouble bottom, GLdouble top, GLdouble near, GLdouble far);



Another Perspective Projection

- gluPerspective (GLdouble fovy, GLdouble aspect, GLdouble near, GLdouble far)
  - fovy -- angle of the field of view in x-z plane (0-180 degree)
  - aspect = w / h;
  - znear, zfar -- the distances between the viewpoint and the clipping planes along the negative z-axis. The should be always positive.



Perspective Viewing with OpenGL

- Positioning and aiming camera
  - glMatrixMode(GL_MODELVIEW);
    glLoadIdentity();
    gluLookAt(eye.x, eye.y, eye.z, // eye position
    look.x, look.y, look.z, // *look at* point
    up.x, up.y, up.z) // up vector
  - Up vector is often set to (0, 1, 0)

Transformation Matrix for LookAt

- Camera coordinate system
  - Axes: **u**, **v**, **n**
    n = *eye – look*
    u = **up** × **n**
    v = **n** × **u**
  - Origin: *eye* (looking in the direction –**n**)
  - If **up** = (0, 1, 0) then
    u = (n_z, 0, $-n_x$) i.e., horizontal
    v = ($-n_x n_y$, $n_x^2+n_z^2$, $-n_z n_y$)
    i.e., more or less upward

Transformation Matrix for LookAt

- Transformation matrix

$$\boldsymbol{V} = \begin{pmatrix} u_x & u_y & u_z & -eye \cdot \mathbf{u} \\ v_x & v_y & v_z & -eye \cdot \mathbf{v} \\ n_x & n_y & n_z & -eye \cdot \mathbf{n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

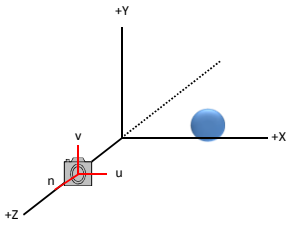
- Verify that ***V*** transformed
  - *eye* to (0, 0, 0, 1)
  - **u** to **x** = (1, 0, 0, 0)
  - **v** to **y** = (0, 1, 0, 0)
  - **n** to **z** = (0, 0, 1, 0)

Example: 3D Viewing

```
glMatrixMode (GL_MODELVIEW);
glLoadIdentity ();
gluLookAt (0, 0, 5, 0, 0, 0, 0, 1, 0);

glTranslatef (3, 0, -2);
glutSolidSphere (1, 10, 10);
```

$$\boldsymbol{V} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



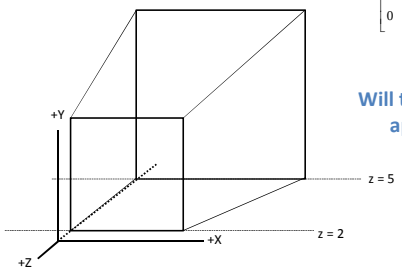
The sphere in viewing coordinates

$$\begin{bmatrix} \boldsymbol{3} \\ \boldsymbol{0} \\ \boldsymbol{-7} \\ \boldsymbol{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{3} \\ \boldsymbol{0} \\ \boldsymbol{-2} \\ \boldsymbol{1} \end{bmatrix}$$

Example: 3D Viewing

```
glMatrixMode (GL_PROJECTION);
glLoadIdentity ();
glFrustum (0, 5, 0, 5, 2, 5);
```

$$M_{pers} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix}$$



Will the sphere appear?

19