King Abdulla II School for Information Technology
Department of Computer Science
Numerical Analysis - Final Exam - Spring 2013

Student Name: Student ID: Section:

Q1 (6 points, 2 each) Given $f(x) = x^3 - x$, approximate $f'(2)$ using:

(a) Central-difference formula $O(h^2)$

(b) Central-difference formula $O(h^4)$

(c) Which of (a) and (b) is more accurate? Explain why?

Q2 (6 points, 2 each) Given $f(x) = x^3 - x$, approximate $\int_1^5 f(x)dx$ with 4 subdivisions using:

(a) Trapezoidal rule

(b) Simpson's rule

(c) Which of (a) and (b) is more accurate? Explain why?

Q3 (6 points) Let

$$A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & -6 \\ 1 & 3 & 2 \end{bmatrix}$$

Construct an upper-triangular form of A using Gaussian elimination with **partial pivoting**.

Q4 (7 points) Use the data linearization method (and change of variable) to find curve fit $y = A\frac{1}{x} + B$ for the three points $(1, 2), (2, 3), (4, 4)$.

Q6 (2 points) Let $(x-2)^2 - lnx = 0$ and $p_0 = 1.5$, Use Newton's method to find p_2.

Q7 (4 points) Show that the matrices A and B are inverses of each other, where

$$A = \begin{bmatrix} 1 & 7 \\ 2 & 5 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

Q8 (6 points, 2 each) Given the linear system $AX = B$ where

$$AX = \begin{bmatrix} \alpha & 0 \\ \beta & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix} = B$$

(a) What α and β values will make the system have no solution?

(b) What α and β values will make the system have infinitely many solutions?

(c) Let $X = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ be a unique solution, determine α and β?

Q9 (6 points) Perform the first three iterations of the Gauss-Seidel method, setting $x_0 = 1, y_0 = 1$ and $z_0 = 1$.

$$\begin{array}{rcrcrcr} x & + & 5y & - & z & = & 8 \\ -9x & + & 3y & + & 2z & = & 3 \\ x & + & 2y & + & 7z & = & 26 \end{array}$$